Bubble in the single best choice for each question you choose to answer.

1. $4^3 \times 4^3 =$
   - (A) 16^9
   - (B) 16^6
   - (C) 4^9
   - (D) 4^6
   - (E) 8^6

     soln When the base is the same, keep the base and add the exponents. □

2. $0.1\% = 1\% -$ _____
   - (A) 0.009%
   - (B) 0.09%
   - (C) 0.9%
   - (D) 10%
   - (E) 99%

     soln $1\% = 0.01$ and $0.1\% = 0.001$ so $x = 1\% - 0.1\% = 0.01 - 0.001 = 0.009 = 0.9\%$ □

3. Of 2009 integers whose product is even, at most _____ can be odd.
   - (A) 2009
   - (B) 2008
   - (C) 2007
   - (D) 1
   - (E) 0

     soln 2008 of them can be odd if one is even because the product of an even integer and any other integer is even. □

4. If x is a natural number, what is the largest possible perimeter of a triangle with side lengths 3, 4, and x?
   - (A) 11
   - (B) 12
   - (C) 13
   - (D) 14
   - (E) 15

     soln The triangle inequality requires each side of a non-degenerate triangle (0 isn't a natural number) to be shorter than the sum of the lengths of the other two sides. □

5. I phoned my mom to help me answer this, the final question on a quiz show: *"How many integers equal their own squares?"* Mom said, "_____." She was right!
   - (A) zero
   - (B) one
   - (C) two
   - (D) three
   - (E) four

     soln $0 = 0^2$ and $1 = 1^2$ □

6. What is the reciprocal of $(\frac{1}{2} \times 4)$?

(A) $2 \times \frac{1}{4}$

(B) $\frac{1}{2} \times 4$

(C) $\frac{1}{2} \times \frac{1}{4}$

(D) 2×4

(E) $2 \times \frac{1}{2}$

SOLN
$$\frac{1}{\frac{1}{2} \times 4} = \frac{1}{\frac{1}{2}} \times \frac{1}{4} = 2 \times \frac{1}{4}$$

Also, the original expression equals 2 and the correct answer equals $\frac{1}{2}$. □

7. If the probability of making a three-point shot in basketball is 0.2, what is the probability of missing a three-point shot?

(A) 0.2

(B) 0.3

(C) 0.4

(D) 0.6

(E) 0.8

SOLN $1 - 0.2 = 0.8$ □

8. Which statement about quadrilaterals is NOT correct?

(A) All rhombuses are parallelograms.

(B) All trapezoids are parallelograms.

(C) All squares are rhombuses.

(D) All squares are rectangles.

(E) All rectangles are parallelograms.

SOLN Trapezoids must have two parallel sides, but the other two don't have to be parallel; in a parallelogram both sets of opposite sides must be parallel. □

9. A *perfect number* is a positive integer which is the sum of its proper positive divisors (that is, excluding the number itself). For example, 28 is a perfect number because $1 + 2 + 4 + 7 + 14 = 28$.
What is the smallest perfect number?

(A) 2

(B) 4

(C) 6

(D) 12

(E) 14

SOLN $1 + 2 + 3 = 6$. We don't know of any odd perfect numbers. □

10. In a scalene triangle, how many angles have the same measure?

(A) 0 (none)

(B) 2

(C) at least 2

(D) 3 (all)

(E) any number

SOLN By definition, a scalene triangle has three sides all of different lengths. Therefore, all of the angles must be of different size (measure). □

11. The price of an item in a store is marked up 60% and then marked down 25%. Which of the following best represents the change from original to final price?

(A) increase by 35%

(B) increase by 45%

(C) increase by 15%

(D) increase by 20%

(E) Depends on the original price

SOLN Let P represent the original price. A 60% markup yields a current price of $1.6P$. Then the markdown of 25% gives a price of $0.75(1.6P) = 1.2P$. The result is a net markup of 20%. □

12. Of the properties commutative, associative, and identity, which hold for the operator $\otimes$ given in the table?

| $\otimes$	a	b	c
a	a	b	c
b	b	a	b
c	c	b	a

(A) commutative and identity only
(B) commutative and associative only
(C) identity and associative only
(D) commutative only
(E) all three

soln The identity is a. The table is symmetric across the main diagonal so the operator is commutative. But it is not associative because
$(b \otimes b) \otimes c = a \otimes c = c$ while
$b \otimes (b \otimes c) = b \otimes b = a.$ □

13. Determine the length of arc AC if $AB = 6$ and O is the center of the circle.

(A) π
(B) 2π
(C) 3π
(D) 4π
(E) 6π
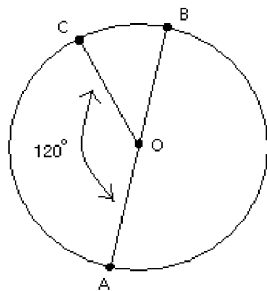
soln The circumference is $\pi D = 6\pi$, but the arc subtended by 120° is 1/3 of the circle, so arc $AC = 2\pi$. □
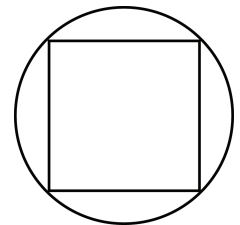
14. Find the median for the set of values.
    3, 13, 4, 1, 4, 6, 7, 1, 5, 1

(A) 1
(B) 3
(C) 4
(D) 4.5
(E) 5

soln List them in ascending order and take the middle value or the average of the two middle values.
1, 1, 1, 3, 4, 4, 5, 6, 7, 13.
The two middle values are 4 and 4. □

15. A square is inscribed inside a circle. The area of the square is what percent of the area of the circle (to the nearest percent)?

(A) 31%
(B) 50%
(C) 64%
(D) 76%
(E) 89%

soln Let the radius of the circle be 1. By the Pythagorean theorem the inscribed square will have sides of length $\sqrt{2}$ and area of 2. The area of the square to the area of the circle will be $2/\pi$ which is pretty close to $2/3 = 0.66\overline{6}\ldots$. This means the correct response must be 64%. □

16. If a steel ball one inch in diameter weighs one pound, how much will a steel ball two inches in diameter weigh?

(A) 2 lbs

(B) 3 lbs

(C) 4 lbs

(D) 5 lbs

(E) 8 lbs

> $\mathcal{SOLN}$ The volume of a sphere scales as the radius cubed ($V = \frac{4}{3}\pi r^3$), so if the radius is doubled the volume is multiplied by $2^3 = 8$, and so is the weight if we assume constant density. □

18. Which equation best represents the graph?

(A) $y = -\frac{2}{5}x - 1$

(B) $y = \frac{2}{5}x - 1$

(C) $y = \frac{5}{2}x - 1$

(D) $y = -\frac{5}{2}x - 1$

(E) $y = \frac{5}{2}x + 1$

> $\mathcal{SOLN}$ The slope is negative and less than one; the intercept is -1.
> (Graph made with Graphing Calculator.) □

17. Pascal's triangle has 1s at each end of each row and the other numbers being the sum of the two immediately diagonally above it.

```
        1              row 0
      1   1            row 1
    1   2   1          row 2
  1   3   3   1        row 3
1   4   6   4   1      row 4
        ⋮
```
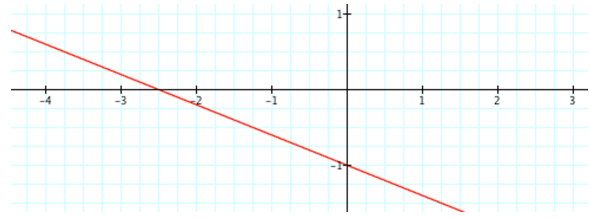
What is the sum of the numbers in row 7?

(A) 64

(B) 65

(C) 66

(D) 67

(E) 128

> $\mathcal{SOLN}$ The long way is to write three more rows in the table and add up the entries in row 7. The short way is to see that the sum of the entries in row n is 2^n. □

19. Solve the inequality.

$$1 < \frac{1}{3 - 2x} < 3$$

(A) $-\frac{3}{4} < x < -\frac{3}{16}$

(B) $\frac{3}{4} < x < 3$

(C) $-\frac{16}{3} < x < -\frac{4}{3}$

(D) $1 < x < \frac{4}{3}$

(E) $-\frac{4}{3} < x < 1$

> $\mathcal{SOLN}$
>
> $$1 < \frac{1}{3 - 2x} < 3$$
>
> $3 - 2x < 1 \quad \text{and} \quad 1 < 3(3 - 2x)$
>
> $2 < 2x \quad \text{and} \quad -8 < -6x$
>
> $1 < x \quad \text{and} \quad \frac{-8}{-6} > x$
>
> $1 < x \quad \text{and} \quad x < \frac{4}{3}$ □

20. A distance runner can run 6 mph downhill, but his speed is cut to 2 mph when he runs uphill. If he runs 6 miles downhill and 6 miles uphill, what is his average speed?

(A) 2 mph

(B) 3 mph

(C) 4 mph

(D) 5 mph

(E) 6 mph

$\boxed{\mathscr{SOLN}}$ The average speed is the total distance divided by the total time. The time downhill is 1 h and the time uphill is 6 miles $\div$ 2 mph $=$ 3 h. So the total time is 4 h. 12 miles $\div$ 4 h $=$ 3 mph. $\qquad\square$